

Announcements

11.03

Translation with Quantifiers

The Step-by-Step Method

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11.03.11

- ① The midterms are graded, they will be handed back on Tuesday
 - Check on grades through Bb this weekend
- ② Additional HW scores should be available too

Outline

- ① Translation Review
- ② Step-by-Step Translation
- ③ Ambiguity

The 4 Aristotelian Forms

Review

The Aristotelian Forms and Their Translations

<i>All A's are B's</i>	$\forall x (A(x) \rightarrow B(x))$
<i>Some A's are B's</i>	$\exists x (A(x) \wedge B(x))$
<i>No A's are B's</i>	$\forall x (A(x) \rightarrow \neg B(x))$
<i>Some A's are not B's</i>	$\exists x (A(x) \wedge \neg B(x))$

Subjects and Objects

Some Terminology

- Some predicates like *love* relate two things:
(1) *Kay loves Jay*
- When you have a predicate that relates two things, it's helpful to have some terminology to distinguish those two things
- Kay* is the **subject**
- Jay* is the **object**
- Intuitively, the subject is what the sentence is primarily about

Roaming Quantifiers

In Object Position

- So far, we've only considered sentences with quantifiers in subject-position:
(2) *Every cube is in front of **b***
- What about when you have a quantifier in object-position?
(3) ***b** is in front of **everything***
- Just stick $\forall$ out in front of the predicate, and 'quantify into' the object position

$\forall x \text{FrontOf}(b, x)$

Roaming Quantifiers

More on Object Position

- Okay, but what happens when the quantifier in object position is **restricted**
(4) ***b** is in front of every **cube***
- You have to **move** its **restrictor** out front **too**:
(4') $\forall x (\text{Cube}(x) \rightarrow \text{FrontOf}(b, x))$
- This holds for **multiply restricted** ones too:
(5) ***b** is in front of **every small cube***
Translates as:
(5') $\forall x ((\text{Cube}(x) \wedge \text{Small}(x)) \rightarrow \text{FrontOf}(b, x))$

Roaming Quantifiers

Some More Examples

- (6) shows that you move the restrictors to the left of the predicate, but no further!
- (6) a. *It's not the case that **b** is a large cube*
b. $\neg \exists y (\text{Large}(y) \wedge \text{Cube}(y) \wedge b = y)$
- (7) a. *It's not the case that something is a large cube*
b. $\neg \exists y (\text{Large}(y) \wedge \text{Cube}(y) \wedge \exists x x = y)$
- (8) a. *Everything between **c** and **b** is **a***
b. $\forall x (\text{Between}(x, c, b) \rightarrow x = a)$
- (9) a. *Everything between **c** and **b** is a cube*
b. $\forall x (\text{Between}(x, c, b) \rightarrow \exists y (\text{Cube}(y) \wedge x = y))$

A Systematic Method For Translating Mixed Quantifiers

- Translating simple quantificational sentences into FOL is hard enough
- Once we consider sentences with multiple and mixed quantifiers, things get even harder
- To address this situation we are going to learn a systematic method for translating quantificational sentences
- We'll first go through an application of the method and then state abstract what the method is

The Method Learning by Example

(10) *Every cube is to the left of a tetrahedron*

- First, note (10)'s **general form**: *Every A is B*
- So, our translation will have the form:
(10') $\forall x [A(x) \rightarrow B(x)]$
- We just need to find $A(x)$ and $B(x)$
- $A(x) = \text{Cube}(x)$, but what is $B(x)$?
- Something like: x *is-to-the-left-of-a-tetrahedron*
 - This predicate translates as: $\exists y (\text{Tet}(y) \wedge \text{LeftOf}(x, y))$
 - So, $B(x) = \exists y (\text{Tet}(y) \wedge \text{LeftOf}(x, y))$
- Finally, we just plug $A(x)$ and $B(x)$ into (10'):
(10'') $\forall x [\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{LeftOf}(x, y))]$

The Method In the Abstract

The Step-by-Step Translation Method

- 1 Determine the general form of the sentence
 - E.g. *Every A is B*, *No A is B*
- 2 Write the *skeleton* for that form
 - E.g. $\forall x [A(x) \rightarrow B(x)]$, $\forall x [A(x) \rightarrow \neg B(x)]$
- 3 Find the parts of the skeleton:
 - E.g. find $A(x)$ and $B(x)$
 - If the parts are complex, start with an informal approximation
 - If the parts themselves contain mixed or multiple quantifiers, repeat this method on them
- 4 Plug the parts into the skeleton

The Method A Second Example

(11) *Some tetrahedron is in front of every small cube*

- 1 The Form: *Some A is B*
- 2 The Skeleton: $\exists x [A(x) \wedge B(x)]$
- 3 Find the parts:
 - $A(x) = \text{Tet}(x)$, what about $B(x)$?
 - $B(x)$ is complex, so informally approximate:
 x *is-in-front-of-every-small-cube*
 - Now translate $B(x)$:
 $\forall y ((\text{Small}(y) \wedge \text{Cube}(y)) \rightarrow \text{FrontOf}(x, y))$

- 4 Fill in the skeleton:

(11') $\exists x [\text{Tet}(x) \wedge \forall y ((\text{Small}(y) \wedge \text{Cube}(y)) \rightarrow \text{FrontOf}(x, y))]$

The Method

How to Write it Down

(11) *Some tetrahedron is in front of every small cube*

- 1 Recognize the form and write the appropriate skeleton:

(11a) $\exists x [A(x) \wedge B(x)]$

- 2 Fill incrementally, using approximation where necessary:

(11b) $\exists x [\text{Tet}(x) \wedge B(x)]$

(11c) $\exists x [\text{Tet}(x) \wedge x \text{ is-in-front-of-every-small-cube}]$

- 3 Once you've arrived at approximations with a single quantifier, translate them and plug them in:

(11') $\exists x [\text{Tet}(x) \wedge \forall y ((\text{Small}(y) \wedge \text{Cube}(y)) \rightarrow \text{FrontOf}(x, y))]$

In Class Exercise

Exercise 11.39

Yet Another Example

A Harder One

(12) *Some cube with nothing in front of it has something in back of it*

- 1 The form is *Some A is B*, so its skeleton is:

(14a) $\exists x [A(x) \wedge B(x)]$

- 2 $A(x)$ and $B(x)$ are complex, so we start with approximations:

(14b) $\exists x [A(x) \wedge \text{something-is-in-back-of } x]$

(14c) $\exists x [A(x) \wedge \exists z \text{ BackOf}(z, x)]$

(14d) $\exists x [(\text{Cube}(x) \wedge \text{nothing-is-in-front-of } x) \wedge \exists z \text{ BackOf}(z, x)]$

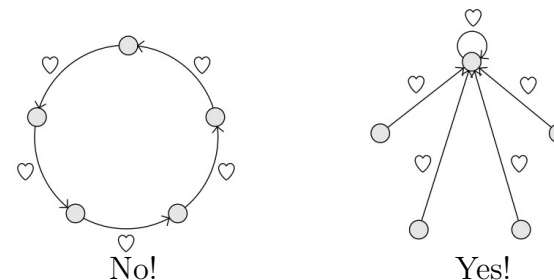
(14') $\exists x [(\text{Cube}(x) \wedge \forall y \neg \text{FrontOf}(y, x)) \wedge \exists z \text{ BackOf}(z, x)]$

The Method

Particularity

(13) *Every one loves a particular someone*

- Which picture does this describe?



- So, we translate (13) as:

(13') $\exists y \forall x \text{ Loves}(x, y)$

The Method

Some Questions

(13) *Every one loves a particular someone*

(13') $\exists y \forall x \text{Loves}(x, y)$

- In (13) the order of appearance is *Universal-Existential*
- In (13') the order is *Existential-Universal*
- What gives?
- The word *particular* signals that the existential takes *widest scope*
- In general $\exists \forall$ sentences describe *some particular thing* being related to *everything*
- $\forall \exists$ sentences describe *every thing* being related to *some thing or other*

The Method

A More Advanced Application

(14) *Every cube is the same size as a particular tetrahedron*

- The form is *Every A is B*, so we start from the appropriate skeleton:

(14a) $\forall x (A(x) \rightarrow B(x))$

(14b) $\forall x (\text{Cube}(x) \rightarrow B(x))$

(14c) $\forall x (\text{Cube}(x) \rightarrow x \text{ the same size as a particular-tet})$

(14d) $\forall x (\text{Cube}(x) \rightarrow \text{SameSize}(x, a\text{-particular-tet}))$

- We know *particular* makes *existentials* take wide scope, so the next step from (14d) is:

(14') $\exists y [\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{SameSize}(x, y))]$

Ambiguity

A Catch

- Our step-by-step method works wonderfully in most cases
- But, there are some things you have to be wary of when translating from English to FOL
- One of them is *ambiguity*

Ambiguity

(15) *Every minute a man is mugged in New York City*

- **Joke:** we are going to meet the poor guy tonight
- We generally interpret (15) as:

(15a) $\forall x [\text{Min}(x) \rightarrow \exists y (\text{Man}(y) \wedge \text{MuggedIn}(y, x, \text{nyc}))]$

 - *For every minute x , there is at least one man y such that y is mugged at x in NYC*
- But the joke plays on the fact that (15) also seems to leave open the interpretation:

(15b) $\exists y [\text{Man}(y) \wedge \forall x (\text{Min}(x) \rightarrow \text{MuggedIn}(y, x, \text{nyc}))]$

 - *There is at least one man y such that for every minute x , y is mugged at x in NYC*

Ambiguity

What it is and Why it Matters

- In general, a sentence is *ambiguous* when it has two or more *different* interpretations
- Sentences involving quantifiers have a preferred interpretation, but a second interpretation is often possible
- Sentences of FOL are *not* ambiguous
- So, when you translate an English sentence into FOL you will sometimes have to think about which possible interpretation of that sentence you should be capturing
- In FOL, these differences generally amount to different quantifier orderings

Ambiguity

Another Example

(16) *Every cube is the same size as a dodecahedron*

- Does (16) say that every cube is the same size as some dodec or other?
- Or does it say that there is a particular dodec which every cube is the same size as?
- Let's solidify the difference between the two claims in Tarski's World